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NONGRAY RADIATIVE TRANSFER IN A TURBULENT GAS LAYER

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Abstract—The energy equation describing nongray radiation transfer and simultaneous turbulent diffusion in a layer of molecular gas enclosed by parallel black walls is developed into a linear integral equation with symmetric kernel. The solutions for temperature and convective and radiative heat flux profiles are strictly valid for constant volume heat source, but are good approximations to those for established turbulent flow. The results show the effect of self-absorption by cold gas in the vicinity of the wall and the coupling between nongray radiation and turbulent diffusion. A correlation of the results in general dimensionless form makes it possible to calculate readily the effects of turbulent boundary layer blockage of nongray gas radiation from any of a number of common gases.

NOMENCLATURE

A, total band absorption [cm^{-1}];
A_s^*, slab band absorptance [dimensionless];
$A_{i,j}$, defined in Appendix;
b, Mei and Squire constant = 3·4;
B, Planck black body spectral radiosity [$W/m^2\,cm^{-1}$];
c_f, skin friction coefficient;
c_p, specific heat at constant pressure [Ws/gmK];
c, speed of light, $c = 2{\cdot}998 \times 10^{10}$ [cm/s];
C^*, channel emittance factor, defined by equation (27);
d, line spacing [cm^{-1}];
D^*, a dimensionless grouping defined by equation (15);
D_h, hydraulic diameter [m];
E_n, exponential integral function of order n;
$F_{i,j}$, defined in Appendix;
h, Planck's constant, $h = 6{\cdot}626 \times 10^{-27}$ [erg-s];
k_m, molecular thermal conductivity [W/mK];
K, Von Karman constant = 0·40;
k, Boltzmann constant, $k = 1{\cdot}3805 \times 10^{-16}$ erg/K;
L, path length [m];
m, total number of subdivisions;
Nu, Nusselt number;
P, dimensionless temperature gradient;
Pr, Prandtl number;
q, heat flux [W/m^2];
$\dot{Q}_v$, volume heat release [W/m^3];
R_{dm}, radiation conductance to molecular conductance parameter, defined by equation (18);
R_t, turbulent Reynolds number;
Re_{D_h}, Reynolds number based on hydraulic diameter;
S^*, zeroth moment of slab band absorptance;
t, optical depth at maximum absorption in band;
T, temperature [K];
T^*, dimensionless temperature, defined by equation (16);
T_v, volume average temperature [K];
V, a constant; also velocity;
W, dimensionless weighting ratio defined by equation (20);
y, distance from wall [m];

y^+, dimensionless distance defined by equation (3);
y^*, dimensionless distance y/δ;
z^*, dimensionless distance transformed to logarithmic scale by equation (17).

Greek letters
α, integrated band intensity [$cm^{-1}/g\,m^{-2}$];
γ, line width, [cm^{-1}]; also Euler–Mascheroni constant = 0·5772156....;
δ, half thickness of gas layer [m];
$\delta_{i,j}$, Kronecker delta;
ε_H, eddy diffusivity of heat [m^2/s];
ε^+, dimensionless eddy diffusivity;
μ, micron, also viscosity [g/m s];
ν, wavenumber;
ν_k, band center or band head of the kth band;
π, 3·1415927....;
ρ, density of gas [g/m^3];
τ, optical depth at maximum absorption in band;
ω, bandwidth parameter [cm^{-1}].

Subscripts
BL, boundary layer;
C, convective;
cl, centerline;
H, band head or band center;
i, ith location;
j, jth location;
k, kth band;
Q, from spectral quadrature;
R, radiative;
s; slab;
T, total;
t, turbulent;
v, volume;
w, wall;
ν, spectral.

Overscore
$-$, average.

INTRODUCTION

THE HEAT transfer to a wall of a furnace combustion chamber from a hot turbulent gas contained within the chamber is usually computed by adding a radiative flux to a convective one. The former is calculated by treating the gas as isothermal at a bulk average temperature with temperature jump between the gas and wall. The latter is found from a convective Nusselt number. In short, the radiative flux is found as though there were infinite convective transport within the gas but none at the wall so that the gas is isothermal, while the convective transport at the wall is found as though the radiation mechanism were inoperative.

Qualitatively it is known that the calculations sketched above are naive, because, in reality, the convective transport lowers the temperature of the gas near the wall, and this cold layer of gas in turn decreases the radiative transport, acting somewhat as a shield against the hot gas radiation. However, the energy deposited in the cold layer should steepen the wall gradient and thus increase the convective transport. The question then arises: How much does a turbulent, cold gas layer decrease radiative transport by absorption, and how much of the absorbed radiation flux reappears at the wall as increased convective transport?

A complicating factor is the nature of molecular gas radiation, which occurs in parts of the spectrum called bands. At spectral positions of the maximum absorption in gas radiation bands, the photon mean free path may be only a millimeter or less, with the result that a cold layer is indeed effective in blocking radiation transfer to the wall. But in the band wings the photon mean free path is very large, at one spectral location large enough for a layer near the wall to be virtually transparent, and, at another further out in the band wing, so large that the entire combustion chamber volume is virtually transparent. The nongray radiative transfer occurs mainly in narrow spectral regions where the wall layer is optically thin or transparent but in which the radiating volume is optically thick.

It is the purpose of this paper to explore the interaction of radiation with turbulent transport. For the sake of simplicity a turbulent gas layer with constant volume heat source is imagined enclosed between two black plane parallel walls. The gas contains nongray bands modeled by exponential decay of the absorption coefficient with spectral position removed from that of maximum absorption.

LITERATURE SURVEY

Previous investigations for the case of pure conduction (or laminar Couette flow) and gray radiation have been reviewed by Cess [1] and Viskanta [2]. For a boundary layer flow Cess argued that "the *maximum* effect that radiation can exert upon the convective process is to reduce ... the Nusselt number from that for a uniform surface heat rate to the value for a uniform surface temperature. If the flow is turbulent, this difference is only about 4 per cent, and it may therefore be concluded that for turbulent flow across a flat plate any radiation effects upon the surface boundary condition will have a slight effect upon the (convective) Nusselt number."

Cess, Mighdoll and Tiwari [3] examined the case of simultaneous radiation and molecular conduction between infinite black parallel plates enclosing a molecular gas with a single exponential winged vibration-rotation band and with a uniform volume heat source. It will be recognized that this physical arrangement is exactly the one analyzed herein, without turbulence. A substitute kernel was employed, and the governing equation was solved by quadratic and quartic colocation for the case of CO gas. A result of significance to the present study was the corroboration of an observation based upon single-line-of-sight calculations [4] that line shape, as manifested by the line width to spacing ratio, was of secondary importance to band shape, as manifested by a band decay spectral width ω or A_0, when the optical depth at the band head or center was large. (This behavior is not because "the wings possess a more continuous structure", attributed by [3] to [4], but because the band absorption $dA/d(\rho L) = \omega/(\rho L)$ is independent of γ/d when ρL is large.)

Reference [3] built upon the work of Gille and Goody [5] and Wang [6] who showed that plane parallel problems with small temperature differences can be formulated in terms of a "modified emissivity", namely the "internal total emissivity" formed by weighting spectral emissivity by the partial derivative of Planck function with respect to temperature.

Greif and Habib [7] solved for temperature profiles and heat transfer in the turbulent channel flow of an optically thin radiating gas with small temperature differences enclosed by black walls. A substitute kernel was used as in the case of Cess *et al.*, and the assumption of small optical depth permitted the simplification of the kernel and facilitated obtaining the solution. Prior studies of turbulent flow in cylindrical ducts were made by Nichols [8] and Landram, Greif and Habib [9].

A slab band absorptance function [10] has been shown to lead to a compact formulation of radiative transfer in a nongray, nonisothermal molecular gas. The blockage effect of a cold boundary layer was evaluated for a simple straight line boundary layer temperature profile, and recommendations based upon prior work [4] were given for laminar channel flow. Blockage by a turbulent boundary layer was not established.

THEORY

Figure 1 shows the physical arrangement postulated. A gas layer of thickness 2δ exists between two plane parallel black walls. A constant volume heat release occurs. An eddy diffusivity for heat ε_H is taken to be given by the Van Driest law of the wall [11].

$$\varepsilon_H = \frac{k_m}{\rho c_p} Pr_m Pr_t^{-1}\{\tfrac{1}{2}[1 + 4K^2 y^{+2} \times (1 - \exp(-y^+/A^+))^2]^{\frac{1}{2}} - \tfrac{1}{2}\}, \quad (1)$$

where K is the Von Karman constant 0·40, and A^+ is 26. Since the object of the investigation is to examine radiation-turbulent-diffusion interactions, the product $Pr_m Pr_t^{-1}$ is taken equal to unity to simplify the expression. This approximation is exact when Pr_m is somewhat greater

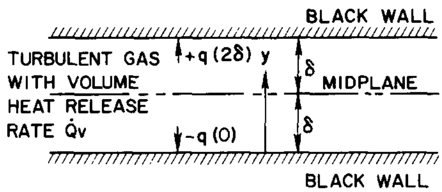

FIG. 1. Turbulent gas layer.

than 0·7 and Pr_t^{-1} is 1·3. By virtue of this simplification

$$\frac{k_m + \rho c_p \varepsilon_H}{k_m} = \varepsilon^+ = \tfrac{1}{2} + \tfrac{1}{2} \times [1 + 4K^2 y^{+2}(1 - \exp(-y^+/A^+))^2]^{\frac{1}{2}}. \quad (2)$$

Mei and Squire [12] account for the approach of ε^+ to $1 + 0{\cdot}09R_t$ in the central regions of the channel by dividing by $1 + by^*$, where $b = 3{\cdot}4$. The quantity y^+ is defined by

$$y^+ = \sqrt{\left(\frac{c_f}{2}\right)}\left(\frac{V\delta\rho}{\mu}\right)\left(\frac{y}{\delta}\right) = R_t y^* \quad (3)$$

where

$$R_t = \sqrt{\left(\frac{c_f}{2}\right)}\frac{V\delta\rho}{\mu} = \frac{1}{4}\sqrt{\left(\frac{f}{8}\right)}Re_{D_h} \quad (4)$$

and

$$y^* = y/\delta. \quad (5)$$

With these relations conservation of energy within the turbulent gas layer takes the form

$$0 = \frac{\mathrm{d}}{\mathrm{d}y}\left(k_m \varepsilon^+ \frac{\mathrm{d}T}{\mathrm{d}y}\right) + \frac{\mathrm{d}}{\mathrm{d}y}(-q_R) + \dot{Q}_v. \quad (6)$$

This relation is subject to boundary conditions

$$T(0) = T(2\delta) = T_w. \quad (7)$$

Integration of equation (6) and the observation that the heat fluxes are zero at $y = \delta$ by virtue of the symmetry shows

$$-k_m \varepsilon^+ \frac{\mathrm{d}T}{\mathrm{d}y} + q_R + \dot{Q}_v(\delta - y) = 0. \quad (8)$$

This expression, since q_R is an integral term shown below, is the governing integro-differential equation for the temperature.

The radiative heat flux in a symmetric channel is given by the following integral terms

$$q_R = \int_0^\infty \{B_{wv} 2E_3(\tau_v) - B_{wv} 2E_3(2\tau_{\delta v} - \tau_v) + \int_0^{\tau_v} 2E_2(\tau_v - t_v)\, B(t_v)\, \mathrm{d}t_v - \int_{\tau_v}^{2\tau_{\delta v}} 2E_2(t_v - \tau_v)\, B(t_v)\, \mathrm{d}t_v\}\, \mathrm{d}v$$

where τ_v is the spectral optical depth at the location where q_R is desired, t_v is any optical depth in the channel, B is the Planck black body spectral radiosity (power per unit area and unit band width), and E_n is the exponential integral function of order n. Allowance for the spectral variation of optical depth introduces great complexity. However, it has been shown that the above expression simplifies for the channel geometry and for many common molecular gases whose spectral variations can be accounted for with exponential-winged bands. From equation (25) of [10]

$$q_R = -\sum_{k=1}^{n} \omega_k \frac{\partial B(v_k, T)}{\partial T} \times \int_0^1 \{A_s^*(\tau_{H,k}(2 - y^* - y'^*)) - A_s^*(\tau_{H,k}|y^* - y'^*|)\} \frac{\mathrm{d}T}{\mathrm{d}y'^*}\, \mathrm{d}y'^*, \quad (9)$$

where A_s^* is the slab band absorptance

$$A_s^*(t) = \ln t + E_1(t) + \gamma + \tfrac{1}{2} - E_3(t), \quad (10)$$

$B(v, T)$ is the Planck radiosity at wavenumber v

$$B(v, T) = \frac{2\pi hc^2 v^3}{e^{hcv/kT} - 1} \quad (11)$$

$\tau_{H,k}$ is the maximum optical depth for the half channel width in the kth absorption band,

$$\tau_{H,k} = \frac{\alpha_k \rho_a \delta}{\omega_k}, \tag{12}$$

where ρ_a is the partial density of the absorbing gas of concern, α_k is the integrated intensity of kth band, and ω_k is the exponential decay width. The quantity y^* is dimensionless distance

$$y^* = y/\delta. \tag{13}$$

Equation (9). as discussed in [10], is appropriate for narrow bands of overlapped lines so that $\partial B(v, T)/\partial T$ does not vary markedly with v near the location v_k of the kth band and is written especially for a flat plate channel formed by two black walls enclosing a gas with temperature symmetric about the midplane and continuous in temperature at the walls. It has also been assumed that the temperature differences are sufficiently small so that the properties α_k, ρ and ω_k and the derivative of the Planck function $\partial B(v_k, T)/\partial T$ do not vary significantly throughout the gas.

Equation (8) can be made dimensionless by dividing by $\dot{Q}_v\delta$. The temperature is likewise rendered dimensionless utilizing $\dot{Q}_v\delta$ divided by a scale conductance $k_m\varepsilon^+(R_t)/\delta$. Further, the length scale, already made dimensionless according to equation (13) can be transformed to a logarithmic coordinate more suitable for turbulent transfer near a wall. The governing integro-differential equation, equation (8), becomes

$$D^* \frac{dT^*}{dz^*} = 1 - y^* - [R_{dm}/\varepsilon^+(R_t)]\, q_R^* \tag{14}$$

where

$$D^* = \frac{\varepsilon^+(R_t y^*)}{\varepsilon^+(R_t)} V e^{-z^*} \tag{15}$$

$$T^* = \frac{T - T_w}{\dot{Q}_v\delta/[k_m\varepsilon^+(R_t)/\delta]} \tag{16}$$

$$y^*(z^*) = \frac{1}{V}[e^{z^*} - 1], \quad z^* = \ln(1 + Vy^*) \tag{17}$$

$$R_{dm} = (\sum_{k=1}^{n} \omega_k B_k' A_k^*)/(k_m/\delta), \quad B_k' \equiv \partial B(v_k, T)/\partial T,$$

$$A_k^* = A_s^*(2\tau_{H,k}) \tag{18}$$

$$q_R^* = \sum_{k=1}^{n} \frac{W_k}{A_k^*} \int_0^1 [A_s^*(\tau_{H,k}(2 - y^* - y'^*)) - A_s^*(\tau_{H,k}|y^* - y'^*|)] \frac{dT^*}{dy'^*} dy'^* \tag{19}$$

$$W_k = \frac{\omega_k B_k' A_k^*}{\sum_{k=1}^{n} \omega_k B_k' A_k^*}. \tag{20}$$

The numerator of equation (18), offset by the denominator of equation (20), is introduced arbitrarily. The quantity R_{dm} can be interpreted physically as a ratio of nongray radiation conductance to molecular conductance. Note that the Rosseland conductivity is not used, because it has no meaning for a nonoverlapped exponential-tailed band. The quantity V is also introduced arbitrarily, but may be set equal to the product of the Von Karman constant and turbulent Reynolds number. The larger is V, the more the region near the wall is stretched by the y^* to z^* transformation.

Equation (14) with equation (19) is a linear integral equation in the temperature gradient dT^*/dz^*. Note that it has a symmetric kernel. The problem of simultaneous nongray radiation and turbulent diffusion has thus been cast into a form characteristic of pure radiation transfer, for example, radiation exchange in an enclosure. As is commonly done in such pure radiation problems, the integral term can be closely approximated by a summation of discrete values, and the problem transformed to solving a set of simultaneous linear algebraic equations. As shown in the Appendix, equation (14) becomes

$$\sum_{j=1}^{m} \{\delta_{i,j} D_i^* + [R_{dm}/\varepsilon^+(R_t)]\, A_{i,j}\} P_j = 1 - y_i \qquad i = 1,2,\ldots, m \tag{21}$$

where P_j is dT^*/dz^* at $z^* = z_j^*$.

RESULTS

Tables 1 and 2 give results for the case of ω_k and $\tau_{H,k}$ invariant with k. Values reported are total, radiative, and convective heat fluxes at the wall made dimensionless by dividing by $(T_v - T_w) k_m/\delta$, where T_v is the volume average temperature. The volume average temperature was used, because it is the temperature appropriate for linearized optically thin radiation. It is also very nearly the bulk average temperature for flow when R_t is large. The centerline temperature is also shown in dimensionless form.

$$Nu_T = \frac{-q(0)\,\delta}{k_m(T_v - T_w)} \tag{22}$$

$$Nu_R = \frac{-q_R(0)\,\delta}{k_m(T_v - T_w)} \tag{23}$$

$$Nu_C = \frac{-q_C(0)\,\delta}{k_m(T_v - T_w)} \tag{24}$$

$$\frac{T_{cl}^*}{T_v^*} = \frac{T(\delta) - T_w}{T_v - T_w}. \tag{25}$$

Figures 2–4 show the temperature profiles between the wall and midplane for three different values of turbulent Reynolds number. The abscissa is $z^*/z^*(y^* = 1)$ in order to stretch the region near the wall. In order to compare the profiles the same value of $V = 40$ was used in all three plots. The Mei and Squire divisor [12] was not included in equation (2) for the results plotted. Its effect is discussed below.

Table 1. Effect of turbulent Reynolds number on dimensionless heat fluxes and center temperature (parameters ω_k and $\tau_{H,k}$ invariant with k)

		$\tau_H = 50$, $A_s^*(2\tau_H) = 5{\cdot}682$			
R_{dm}/A_s^*		$R_t = 100$	$R_t = 300$	$R_t = 1000$	$R_t = 3000$
0·0	Nu_T	8·45	19·59	53·8	140·2
	Nu_R	0·0	0·0	0·0	0·0
	Nu_C	8·45	19·59	53·8	140·2
	T_{cl}^*/T_v^*	1·1480	1·0948	1·0706	1·0595
0·1	Nu_T	8·84	20·01	54·2	140·6
	Nu_R	0·37	0·42	0·46	0·48
	Nu_C	8·47	19·59	53·7	140·1
	T_{cl}^*/T_v^*	1·1486	1·0952	1·0707	1·0595
1·0	Nu_T	12·52	23·8	58·2	144·7
	Nu_R	3·68	4·15	4·6	4·8
	Nu_C	8·84	19·65	53·6	139·9
	T_{cl}^*/T_v^*	1·1542	1·0986	1·0721	1·0601
10·0	Nu_T	48·6	61·1	97·4	185·4
	Nu_R	36·6	40·6	45·0	47·9
	Nu_C	12·0	20·5	52·4	137·5
	T_{cl}^*/T_v^*	1·1883	1·1253	1·0848	1·0653
100·0	Nu_T	399·8	417·0	467·7	575·9
	Nu_R	371·0	384·7	416·6	454·0
	Nu_C	28·8	32·3	51·1	121·9
	T_{cl}^*/T_v^*	1·2409	1·2046	1·1512	1·1039

DISCUSSION

A number of interesting features emerge from the solutions. In Table 1 it is seen that radiation does contribute very markedly to the total transfer when the radiation-conduction-molecular-diffusion-conductance ratio R_{dm} is large. The radiation contribution grows nearly linearly with R_{dm}, and at high values of R_{dm} it dominates

Table 2. Effect of maximum optical depth on dimensionless heat fluxes and center temperature (parameters ω_k and $\tau_{H,k}$ invariant with k)

		$R_t = 1000$			
R_{dm}/A_s^*		$\tau_H = 5$ $A_s^*(2\tau_H) = 3{\cdot}380$	$\tau_H = 10$ $A_s^*(2\tau_H) = 4{\cdot}073$	$\tau_H = 100$ $A_s^*(2\tau_H) = 6{\cdot}376$	$\tau_H = 1000$ $A_s^*(2\tau_H) = 8{\cdot}678$
0·0	Nu_T	53·8	53·8	53·8	53·8
	Nu_R	0·0	0·0	0·0	0·0
	Nu_C	53·8	53·8	53·8	53·8
	T_{cl}^*/T_v^*	1·0706	1·0706	1·0706	1·0706
0·1	Nu_T	54·1	54·1	54·2	54·2
	Nu_R	0·32	0·37	0·48	0·51
	Nu_C	53·8	53·7	53·7	53·7
	T_{cl}^*/T_v^*	1·0706	1·0707	1·0708	1·0708
1·0	Nu_T	56·9	57·4	58·3	58·6
	Nu_R	3·2	3·7	4·8	5·1
	Nu_C	53·7	53·7	53·5	53·5
	T_{cl}^*/T_v^*	1·0710	1·0714	1·0723	1·0726
10·0	Nu_T	85·4	90·1	99·1	101·2
	Nu_R	31·8	36·8	47·1	49·4
	Nu_C	53·6	53·3	52·0	51·7
	T_{cl}^*/T_v^*	1·0754	1·0784	1·0865	1·0887
100·0	Nu_T	368·2	411·0	478·5	490·4
	Nu_R	313·8	357·8	427·9	439·5
	Nu_C	54·4	53·2	50·6	50·9
	T_{cl}^*/T_v^*	1·1042	1·1216	1·1574	1·1644

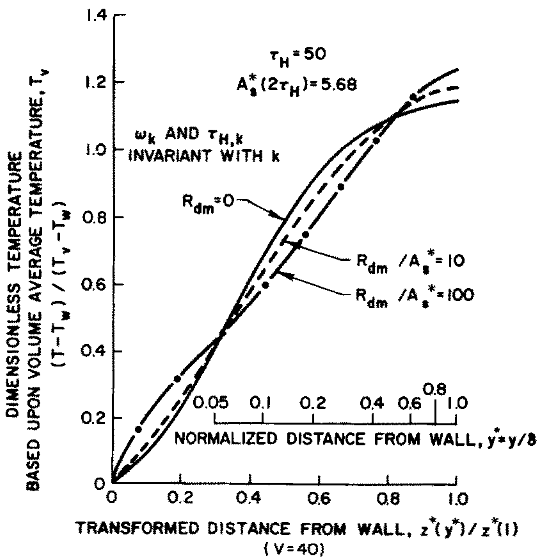

FIG. 2. Temperature profiles for low turbulence ($R_t = 100$).

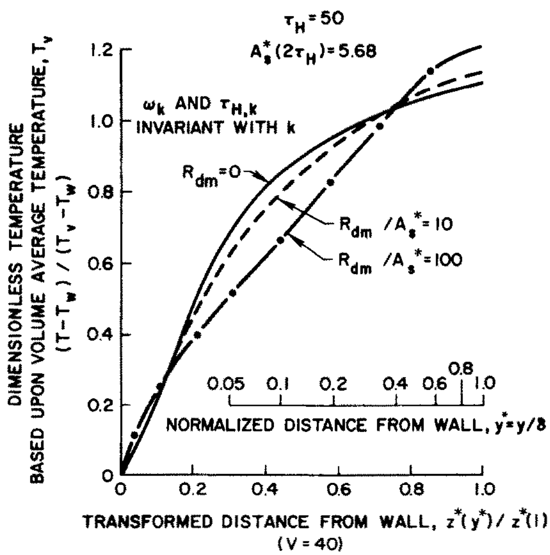

FIG. 3. Temperature profiles for moderate turbulence ($R_t = 300$).

the total transfer. The radiation deposited in the cold wall region does increase the convective transport significantly at the lower values of R_t, but at the higher values the convective Nusselt number based upon volume average temperature actually appears to decrease somewhat with increasing radiation. Increasing turbulance, measured by R_t, increases markedly the convective transport at the wall, but has a lesser effect on the radiative transport, which rises significantly, but at a much less rapid rate.

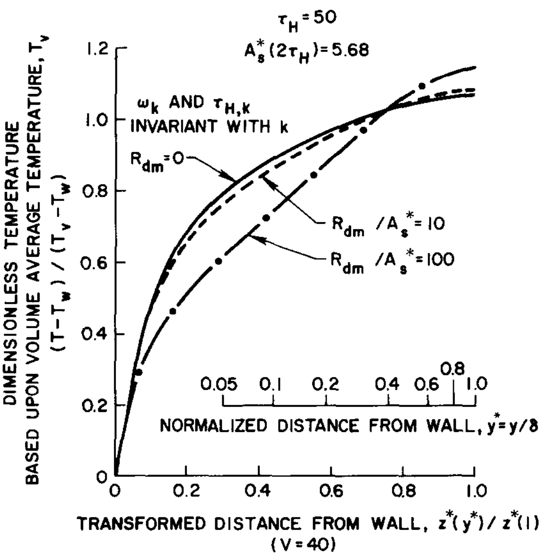

FIG. 4. Temperature profiles for high turbulence (R_t = 1000).

Table 2 shows that the radiation transfer rises rapidly at first as τ_H increases but then at a slow rate when τ_H becomes large. Unlike the case for a gray medium, the nongray gas radiation does not go to a maximum and then decrease with increasing τ_H, but continues to increase slowly.

The figures show that increasing turbulence causes a blunter profile with sharper gradients near the wall. The effect of radiation is to make the temperature profiles less blunt, however. Because the plots are based upon equal volume average temperature, as the profiles become less blunt the midplane temperature must rise to maintain the equality. But the centerline temperature rises only slowly compared to the great increase in wall transfer, as may be seen in the tables. Recall that the region near the wall is stretched and that near the center compressed when z^* is used.

The trends noted in the data suggest a simple correlation good for engineering estimates when detailed calculations are not warranted. At the larger values of R_t, it appears that the usual engineering approximation of merely adding radiative and convective fluxes at the wall is of value. However, the effective emissivity of the gas is reduced by the presence of the temperature gradient near the wall, that is, the effective emissivity is not as large as the isothermal gas emissivity. To account approximately for this latter factor, one regards the core of the gas as isothermal and a linear temperature gradient existing in a layer δ_{BL} thick near the wall. The effective emissivity of such a temperature profile was developed by [10]. It is consequently proposed that

$$Nu_T \doteq Nu_C + R_{dm} \sum_{k=1}^{n} (W_k/A_k^*)\, C^*(\tau_{H,k}, \tau_{BL,k}) \tag{26}$$

where

$$C^*(\tau_{H,k}, \tau_{BL,k}) = \frac{1}{\tau_{BL,k}} [S^*(2\tau_{H,k}) - S^*(2\tau_{H,k} - \tau_{BL,k}) - S^*(\tau_{BL,k})] \tag{27}$$

$$\tau_{BL,k} = \frac{\alpha_k \rho \delta_{BL}}{\omega_k}. \tag{28}$$

The quantity S^* is the zeroth moment of A_s^* developed in [10]. The Appendix contains the explicit formula for it. For ω_k and $\tau_{H,k}$ invariant with k, equation (26) reduces to

$$Nu_T \doteq Nu_C + (R_{dm}/A_s^*(2\tau_H))\, C^*(\tau_H, \tau_{BL}). \tag{29}$$

The thickness of the region near the wall δ_{BL} would be expected to decrease with increasing R_t and increase with R_{dm} (which has a Prandtl number-like effect on thickening the wall layer).

Recall that $Pr_m Pr_t^{-1} = 1{\cdot}0$ was the case investigated. The correlation found was

$$\delta_{BL}^* = \frac{\delta_{BL}}{\delta} = 0{\cdot}66\, R_t^{-0{\cdot}4}[1 + (R_{dm}/A_s^*)]^{0{\cdot}08}. \tag{30}$$

The correlation represented by equations (29) and (30) was found to fit the 32 calculated points in Tables 1 and 2 within 2·6 per cent rms. The worst discrepancy was 6·9 per cent in one case.

To put the values of R_t and Nu in perspective, we may compare them with turbulent channel flow. Of course, in channel flow the temperature T is a function of both a streamwise variable x or z and a transverse variable y or r, and no volume heat source truly exists. However, Thorsen [13] has shown for pipe flow that neglecting the x-variation of T introduces only a one per cent error in predictions of the temperature field, even when radiation is a dominant mode of heat transfer. Furthermore, the streamwise convective term $\rho c_p V \partial T/\partial x$ plays the role of a nearly-constant volume heat source at large values of Re_{D_h} and at values of x beyond the thermal entrance region. Thus we expect that Nu from the present analysis for $R_{dm} = 0$ will be in good agreement with, for example, the Dittus–Boelter equation for turbulent flow, when Re_{D_h} is greater than approximately 10^4.

$$Nu_{D_h} \doteq 0{\cdot}023\, Re_{D_h}^{0{\cdot}8} Pr^{0{\cdot}33}$$

$$Nu_\delta \doteq \tfrac{1}{4} Nu_{D_h} = 0{\cdot}00575\, Re_{D_h}^{0{\cdot}8} Pr^{0{\cdot}33}.$$

In order to make the comparison R_t is related to Re_{D_h} by equation (4)

$$R_t = \sqrt{\left(\frac{c_f}{2}\right)} Re_\delta = \tfrac{1}{4}\sqrt{\left(\frac{c_f}{2}\right)} Re_{D_h}.$$

When the skin friction factor vs Re_{D_h} is taken from Schlichting [14], it is found that $R_t = 100$ corresponds to $Re_{D_h} = 6000$, $R_t = 300$ to $Re_{D_h} = 21\,000$, and $R_t = 1000$ to $Re_{D_h} = 82\,000$. At these values and for $Pr = 1$ the Dittus–Boelter equation yields $Nu_\delta = 6$, 17 and 49 respectively, compared to 8·4, 19·6 and 53·8 respectively from Table 1 for $R_{dm} = 0$. The expected approximate correspondence is exhibited.

As a further aid to putting the values in perspective, a gas such as CO_2 may be considered. At a temperature of 1000°K and for a layer of half width $\delta = 50$ cm, the quantity R_{dm} is approximately 569, using values for 6 bands given in [15]. The A/A_Q correction recommended there was not applied; thus the calculated values of R_{dm} are somewhat low. Of course, the values of $\tau_{H,k}$ and ω_k vary considerably with $k = 1{,}6$ for this real gas, and not all the bands have overlapped lines. For the strongest band at 4·3 μ, $\tau_H \doteq 833$, and for the other major contributor to R_{dm} at 2·7 μ, $\tau_H = 17{\cdot}3$. Another point of interest is that a Re_{D_h} of 82 000 ($R_t = 1000$) corresponds to a velocity of approximately 3 m/s.

As seen above for the case of CO_2, polyatomic gases have a number of absorption bands, and while the several values of ω_k may often be nearly equal, the values of $\tau_{H,k}$ seldom are. Furthermore, the relative values of $\tau_{H,k}$ vary markedly with temperature. This multiplicity of possible values makes it difficult to parameterize the problem. However, it is felt that equations (26) and (30) will give adequate results for engineering estimates.

In order to make an approximate calculation, which can be accomplished even by hand, values of ω_k, $\tau_{H,k}$ and ν_k from [15] are used. The quantity R_{dm} is evaluated according to equation (18) and, the value of R_t is found as explained below equation (30). These values together with a W_k-weighted average $\bar{A}_s^*$ are used in equation (30) to find δ_{BL}. Then the actual values of ω_k and $\tau_{H,k}$ and the value of δ_{BL} are used in equation (27) and (28) to find the values needed for equation (26). This procedure is expected to work well when the gas emission is dominated by one or more strong bands with $\tau_{H,k} > 5$, since the correlation equation (30) was verified for values of $\tau_H > 5$. This expectation is borne out by the values shown in Table 3.

The results of detailed machine calculations based upon equation (21) and the simple correlation calculations agree well, particularly for $R_t \geqslant 300$.

Note that Nu_R divided by R_{dm} is a number less than one which represents the transmission factor for the cold boundary layer. The difference between one and the transmission factor may be thought of as the attenuation or blockage of the hot gas radiation by the cold boundary layer. The magnitude of the attenuation may be seen to be as large as 50 per cent in Tables 2 and 3.

In order to evaluate the effect of center-channel eddy diffusivity upon the transfer rates, the results in Tables 1 and 2 were computed applying the Mei and Squire [12] divisor to equation (2). The Mei and Squire divisor leaves the wall values of eddy diffusivity unchanged but reduces the centerline value of ε^+ to nearly $1 + 0{\cdot}09\ R_t$. This value is only 23 per cent as large as that obtained by using the Deissler expression without modification.

The results obtained for Nu_R with the Mei and Squire divisor were decreased only 5 per cent from those without it. Values of Nu_C were reduced somewhat more, particularly at low R_t. These values were 12–26 per cent lower at $R_t = 3000$ and 100 respectively. The fact that the values of Nu_R were affected but little strengthens confidence in the correlation, equation (30), for the radiative transfer through a turbulent wall layer. It is felt that the correlation could be applied to circular ducts as well, by using τ_H equal to $\frac{1}{4}$ of that obtained based upon the mean beam length, since the geometric mean

Table 3. Comparison of exact results with correlation. CO_2 gas at P = 1 atm

Gas temp.	Channel half thickness	Calculated values of		Turbulence parameter	Exact result	Correlation result
T (°K)	δ (m)	$\bar{A}_s^*$	R_{dm}	R_t	Nu_R	Nu_R
500	0·10	6·46	17·5	100	10·2	10·2
				300	11·5	11·3
				1000	12·8	12·3
				3000	13·8	13·2
	0·50	7·36	119·	100	65·3	62·7
				300	70·7	68·8
				1000	79·0	73·6
				3000	86·4	79·1
1000	0·10	5·01	78·9	100	56·5	54·1
				300	60·2	58·5
				1000	64·7	62·6
				3000	68·0	65·8
	0·50	6·10	569·	100	363·	326·
				300	375·	372·
				1000	402·	403·
				3000	436·	427·
1500	0·10	4·08	132·	100	106·	99·2
				300	110·	106·
				1000	116·	112·
				3000	120·	117·
	0·50	5·09	1037·	100	756·	675·
				300	768·	743·
				1000	800·	795·
				3000	848·	842·

beam length for a channel is four times the channel half-width.

A final point deserving discussion may be the validity of the exponential band representation of molecular gases. Comparison of values of band absorption from single-line-of-sight observations in isothermal gases has indicated a probable accuracy of ± 10 per cent [15]. Emission along a single line of sight in a non-isothermal gas has also been found to be in good agreement [16], when a scaling scheme was used to account for variations in α_k and ω_k [16–18]. Discrepancies were found to be on the order of 10–20 per cent. As far as the ability of the band model to predict correctly the divergence of q_R so that temperature profiles are accurate, only the indirect evidence of the agreement obtained by Schimmel, Novotny and Olsofka [19] is available.

SUMMARY AND CONCLUSIONS

Simultaneous turbulent diffusion in a layer of gas enclosed by parallel black walls was solved for the case of exponential-winged bands with overlapped lines. The results found were well correlated by equations (26) and (30). These equations are sufficiently simple in application that hand calculations may be made. For the conditions of large R_{dm} and moderate R_t typical of industrial furnace operation, the radiation transport was found to be the dominant mode of transport at the wall, but the turbulence played an important part in thinning the cold layer of gas near the wall so that the radiation was not unduly attenuated. Attenuation of radiation by the cold wall layer was found to range up to 50 per cent, when τ_H and R_{dm} were large at $R_t = 1000$.

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APPENDIX

Quadrature for Radiant Flux

In order to obtain a discrete ordinate representation for the energy equation it is necessary to divide the z^* scale into a number of points z_i^* and represent the integral over dT^*/dz'^* as a weighted sum of the temperature gradients at those points. Let

$$z_i^* = (i-1)\Delta z^*,$$

where

$$\Delta z^* = \frac{1}{(m-1)} \ln(1+V),$$

and

$$P_i = (dT^*/dz^*)_{z^* = z_i^*}.$$

Denote a term in the summation over all bands in equation (19) as

$$q_{R,k}^* = \int_0^1 [A_s^*(\tau_{H,k}(2-y^*-y'^*)) - A_s^*(\tau_{H,k}|y^*-y'^*|)] \times \frac{dT^*}{dz'^*}\frac{dz'^*}{dy'^*} dy'^*.$$

Let the integral be replaced by a sum, using a modified trapezoidal rule for simplicity,

$$q_{R,k}^* = \sum_{j=2}^{m} \left\{ \int_{y_{j-1}^*}^{y_j^*} [A_s^*(\tau_{H,k}(2-y_i^*-y'^*)) - A_s^*(\tau_{H,k}|y_i^*-y'^*|)] \, dy'^* \cdot \frac{1}{2}\left[P_{j-1}\left(\frac{dz^*}{dy^*}\right)_{j-1} + P_j\left(\frac{dz^*}{dy^*}\right)_j\right]\right\}$$

The remaining integral terms may be evaluated in terms of the zeroth moment of the slab band absorptance, derived previously [10]

$$S^*(t) = \int_0^t A_s^*(t')\,dt' = t[\ln t + \gamma - \tfrac{1}{2}] + [1 - E_2(t)] - [\tfrac{1}{3} - E_4(t)].$$

Changing variable of integration from y'^* to $(2 - y_i^* - y'^*)$ for the first part and to $(y_i^* - y'^*)$ or $(y'^* - y_i^*)$ for the second part gives

$$F_{i,j,k} = \int_{y_{j-1}^*}^{y_j^*} [A_s^*(\tau_{H,k}(2-y_i^*-y'^*)) - A_s^*(\tau_{H,k}|y_i^*-y'^*|)]\,dy'^*$$
$$= \frac{1}{\tau_{H,k}}\left\{[S^*(\tau_{H,k}(2-y_i^*-y_{j-1})) - S^*(\tau_{H,k}(2-y_i^*-y_j^*))] - \left|S^*(\tau_{H,k}|y_j^*-y_i^*|) - S^*(\tau_{H,k}|y_{j-1}^*-y_i^*|)\right|\right\}.$$

Denoting

$$C_j = \left(\frac{dz^*}{dy^*}\right)_j$$

and collecting like terms gives

$$q_R^* = \sum_{j=1}^{m} A_{i,j} P_j$$

where

$$A_{i,j} = \tfrac{1}{2}(F_{i,j} + F_{i,j+1})C_j \qquad j = 2, 3 \ldots, m-1$$

$$A_{i,1} = \tfrac{1}{2} F_{i,2} C_1$$

$$A_{i,m} = \tfrac{1}{2} F_{i,m} C_m$$

$$F_{i,j} = \sum_{k=1}^{n} W_k F_{i,j,k}/A_k^*.$$

When the expression for q_R^* is substituted into equation (14), equation (21) results, where $\delta_{i,j}$ is understood to be unity for $i = j$ and zero otherwise.

TRANSFERT PAR RAYONNEMENT NON GRIS DANS UNE COUCHE GAZEUSE TURBULENTE

Résumé—L'équation d'énergie décrivant le transfert par rayonnement non gris et la diffusion simultanée par turbulence dans une couche de gaz entourée de parois noires parallèles est développée en une équation intégrale linéaire avec un noyau symétrique. Les solutions pour les profils de température et de flux thermique par convection et rayonnement sont strictement valables pour une source de chaleur à volume constant, mais constituent une bonne approximation de celles obt nues pour un écoulement turbulent établi. Les résultats montrent l'effet de la self-absorption par un gaz froid au voisinage de la paroi et le couplage entre le rayonnement non gris et la diffusion turbulente. Une relation entre les résultats, en général sous forme adimensionnelle, rend possible le calcul rapide des effets d'écran que joue la couche limite turbulente au rayonnement d'un gaz non gris émis par un gaz commun.

WÄRMEÜBERGANG DURCH NICHT GRAUE STRAHLUNG IN EINER TURBULENTEN SCHICHT

Zusammenfassung—Die Energiegleichung, die den Wärmeübergang durch nicht-graue Strahlung und gleichzeitige Diffusion in einer Gasschicht zwischen zwei parallelen schwarzen Wänden beschreibt, wird in eine lineare Integralgleichung mit symmetrischem Kern umgeformt. Die Lösungen für die Temperatur-, Konvektions- und Wärmestrahlungsprofile sind für konstante Wärmequellendichte streng gültig; sie sind gute Näherungen für jene mit ausgebildeter turbulenter Strömung. Die Ergebnisse zeigen den Effekt der Selbstabsorption bei einem kalten Gas in der Nähe der Wand und die Kopplung zwischen nicht-grauer Strahlung und turbulenter Diffusion. Die Korrelation der Ergebnisse in allgemeiner, dimensionsloser Form macht es möglich den Effekt der Blockierung der nicht grauen Strahlung durch turbulente Grenzschichten bei einem beliebigen Gas aus einer Reihe gewöhnlicher Gase einfach zu berechnen.

ЛУЧИСТЫЙ ПЕРЕНОС В ТУРБУЛЕНТНОМ СЛОЕ НЕСЕРОГО ГАЗА

Аннотация—Уравнение энергии, описывающее лучистый перенос с одновременной турбулентной диффузией в слое несерого молекулярного газа, огражденного параллельными черными стенками, преобразуется в линейное интегральное уравнение с симметричным ядром. Решения для профилей температуры, конвективного и лучистого тепловых потоков строго справедливы для источника тепла постоянного объёма и являются хорошими приближениями в случае развитого турбулентного потока. Полученные результаты вскрывают эффект самопоглощения холодным газом вблизи стенки и связь несерого излучения и турбулентной диффузии. Корреляция резудьтатов в общей безразмерной форме позволяет определить эффекты блокирования излучения турбулентного пограничного слоя несерого газа на примере любого обычного газа.